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Compressibility and structure of turbulence in supersonic shear flows

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Abstract – In this paper, different aspects of supersonic turbulent flows are discussed, in connection with some properties of compressible turbulence. The discussion is based on experiments, in which particular features have been found. The first point is the influence of density gradients. It is shown that changing the density gradient in a boundary layer does not change the particularly small size of the energetic scales. This effect is attributed to compressibility and not to density stratification, probably by modifying the 'inactive motions'. The second point is about the importance of acoustics in a mixing layer. The results show that turbulent transport of momentum and heat follows the same analogy as in boundary layers, and that the energy losses by acoustic radiation are too weak to explain the reduction of the spreading rate, which is the major global effect of compressibility. © 2000 Éditions scientifiques et médicales Elsevier SAS

compressible turbulence / supersonic flows / boundary layer / mixing layer

1. Introduction

Important developments in the study compressible turbulence have been achieved in the last decade, by theoretical and numerical works. New properties have been displayed, for example the peculiar shape of large eddies, and the possible importance of dilatation dissipation and pressure-divergence terms has been much discussed. However, the applicability of such results to supersonic turbulent shear flows is not always well defined. In particular, the definition of the different regimes, which can be expected, and the relevant Mach numbers to be used are variables, which are not totally defined. Some attempts have been made to discuss the definition of the pertinent Mach numbers to characterize compressibility effects on turbulence (see for example Smits and Dussauge [1]), but no real comparison with experiments has been performed. Reviews have been published which discuss the properties of turbulence in particular flows. For example, Spina, Smits and Robinson [2] and more recently Dussauge et al. [3] have given an extensive analysis of the experimental results in supersonic boundary layers. One of their conclusions is that the differences in the structure of turbulence between supersonic and subsonic layers are rather small, probably depending on the intermittency factor, on the fractal dimension of the instantaneous edge, or on the longitudinal integral scale. This can be understood as a confirmation (if needed) of the classical Morkovin's hypothesis [4]. For Mach numbers of less than 5, and as long as we consider properties related to the level of energy, boundary layers behave as flows with variable fluid properties, and for many purposes, it is sufficient to assume that the effects of compressibility are low.

On the other hand, supersonic mixing layers can perhaps be considered as the archetype of turbulent flows in which compressibility is important: as shown by Papamoschou and Roshko [5], the spreading rate depends

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on Mach number, and is, in a way, independent of transverse density gradients. Considerable efforts have been made in the last decade to explain this effect, but the reason for such a decrease is still not totally identified. Vreman, Sandham, Luo [6] have proposed to relate it to a decrease in the production of turbulence. There are several other likely candidates to explain the observed effects, such as a modification of the anisotropy of the Reynolds stresses related to an alteration of the structure of the pressure, an enhancement of dissipation through dilatation dissipation in Mach waves or shock waves produced by eddies (Husssaini, Collier and Bushnell's 'eddy shocklets', [7]), or the loss of energy by acoustic radiation. However, none of these hypotheses has been conclusively checked by experiments, and results on this side are still badly needed.

Recently, the question of the integral scale of turbulence has been reconsidered. Dussauge and Smits [8] have shown that the integral scale and the production scale based on longitudinal velocity fluctuations in supersonic boundary layers are the half of their subsonic counterparts. It was hypothesized that this could be related to a modification of the shape of the large eddies or to a modification of the inactive motions. However, it was not clear if this effect should be attributed to density gradient or to Mach number. More recent works (Debiève et al. [9] and Muscat [10]) have shown that the same result is found in a supersonic mixing layer, at a convective Mach number of 0.61. As compressibility and density effects are present in the mixing layer, and as compressibility is supposed to have little influence on boundary layers, it was expected that the reduction of the energetic scales could be attributed to the density gradient.

The objective of this paper is to bring some new experimental information on these questions. Two aspects will be examined. The first is the influence of density gradient on the scales of turbulence and on the shape of the spectra. This was the objective of an experiment on a boundary layer on a heated wall. The second is in connection with the possible role of acoustics in compressible turbulence. If the properties of supersonic mixing layers can be attributed to a modification of the role of pressure, two consequences may be expected, and will be tested in an experiment at a convective Mach number of 0.61. Firstly, the analogy between momentum and heat diffusion, which are efficient for boundary layers, may be altered; verifying from measurements if classical analogies still apply can be an indication about the role of pressure fluctuations. Secondly, energy can be lost by acoustic radiation. An estimate of this energy loss will be made from radiated pressure measurements, and will be given at the end.

2. Turbulence spectra in a supersonic boundary layer

2.1. Introduction, experimental conditions

An experiment was conducted in the continuous supersonic wind tunnel of Institut de Recherche sur les Phénomènes Hors Equilibre. Measurements were performed in two flow conditions. In a first series of measurements, a supersonic boundary layer on a flat adiabatic plate was explored. In a second step, the wall was heated. Density ratios between the external flow and the wall, which would be 1 at low speeds, was 1.9 in the adiabatic case, and 3.8 for the maximum heating of the plate.

The nominal conditions of the flow are as follows. A fully developed boundary layer flow at a Mach number $M = 2.3$ has been installed. In the outer flow, the stagnation pressure was $p_{\mathrm{te}} = 0.5$ atm, and the stagnation temperature was $T_{\mathrm{te}} = 300$ K. The wall temperature T_{w} was $T_{\mathrm{w}}/T_{\mathrm{r}} = 1$ or 2, where T_{r} is the adiabatic recovery temperature. The measurements reported in this paper were performed at a distance larger than 30δ downstream of the beginning of plate heating, where δ is the thickness of the boundary layer at the beginning of the test section, $\delta = 0.8$ cm. Note that in spite of the high temperature of the heated wall ($T_{\mathrm{w}} \approx 600$ K), as the acceleration terms are large, buoyancy effects are negligible.

The turbulent signal was obtained from a Constant Current hotwire Anemometer DISA type 56C02. The bandwidth of the uncompensated amplifier was 200 kHz. The probes were made of a tungsten wire of 5 μm in diameter. The aspect ratio of the wires was 200.

Spectral analysis was applied to the signals. The technique of Kovasznay's fluctuation diagram [11] was used to separate the temperature from velocity fluctuations, by varying the overheating of the wire. The technique was applied for each frequency of the spectrum, as done for example by Morkovin [4] or Demetriades [12] and provided frequency spectra of velocity and of temperature separately. Taylor's hypothesis has been used to derive wave number spectra. Flow arrangements and a description of the experimental technique are given in full detail in Laurent [13].

2.2. Results

Most of the present measurements were taken at the external edge of the law of the wall. A first point was to examine the spectra of the different quantities available by CCA (Velocity, temperature, mass flux, total temperature), to check if they involve the same spectral domains. This is motivated by two reasons.

The first one is the comparison between spectra of temperature and velocity. It is known that in subsonic boundary layers, temperature and longitudinal velocity spectra are different: temperature fluctuations contain higher frequencies (see for example Fulachier [14]), and that in adiabatic supersonic layers, according to the

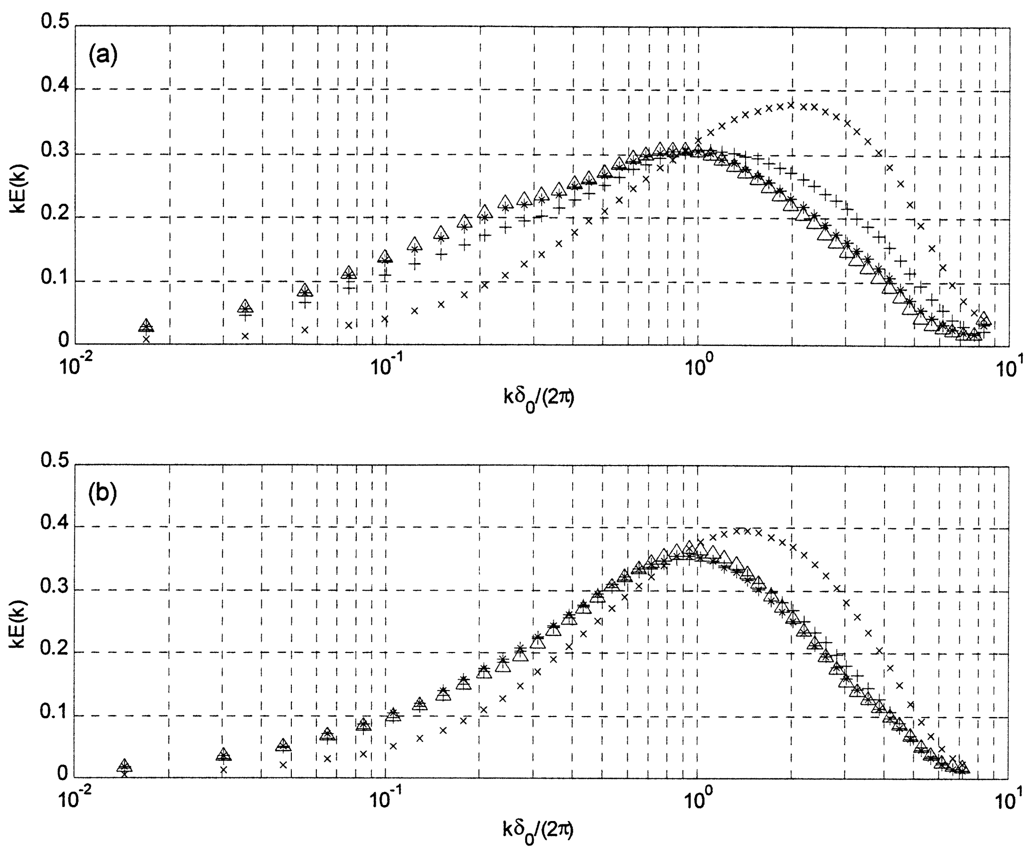

Figure 1. Power spectral densities: Δ velocity; $*$ momentum; $+$ temperature; $\times$ total temperature; (a) adibatic case; (b) heated case.

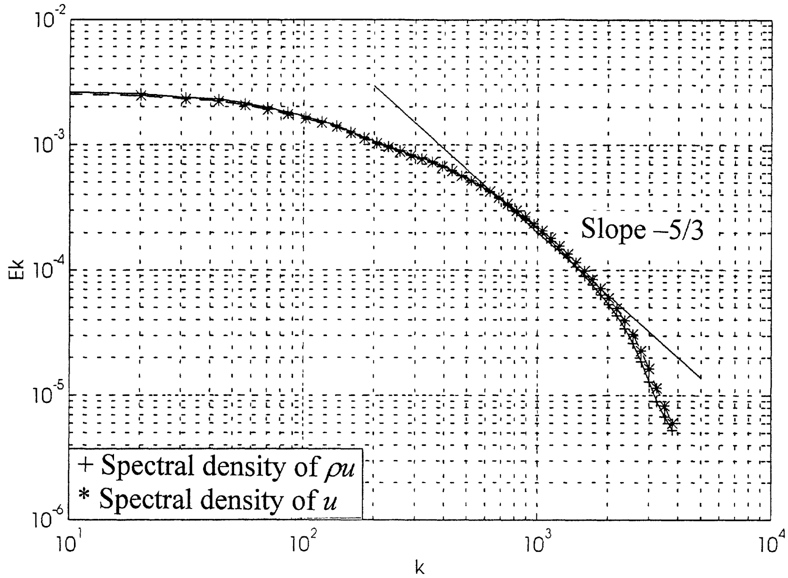

Figure 2. Comparison of spectral densities of u and ρu. Adiabatic case, $y/\delta = 0.2$.

Strong Reynolds Analogy (SRA), spectra of temperature and of velocity have shapes rather similar. In subsonic flow, this frequency shift can be explained from the equality of spectra of turbulent kinetic energy and of temperature (Fulachier [14]). As vertical velocity fluctuations contain higher frequencies than the longitudinal ones, this can explain that the temperature spectrum contains higher frequencies, too. In the case of supersonic boundary layers, an example is given in *figure 1*, which shows that the stagnation temperature, and to a much lesser extent static temperature, have fluctuations ranging at higher frequencies than the fluctuations of velocity or of mass flux. This difference in the frequency ranges was found to decrease in the external boundary layer. Now, when the density ratio is increased by the wall heating ($T_w/T_r \approx 2$) with the same external Mach number, about the same shift is still observed between velocity and total temperature (*figure 1(b)*). Therefore, this is a trend common to the three cases of subsonic, heated and non-heated supersonic boundary layers.

The second reason is the interpretation of the measurements performed with Constant Temperature Anemometers (CTA). This is probably the most common apparatus used in supersonic flow for turbulence measurements. In the appropriate operating conditions, the CTA delivers a signal proportional to mass flux $(\rho u)'$. Comparisons of spectra are made in *figures 1* and *2*. They show that spectra of velocity and of mass flux are close to each other, even with a wall heat flux. There is a simple situation for which this result can be explained, from SRA arguments. For a flow with zero stagnation temperature fluctuations, the fluctuating mass flux can be linearized and expressed as a function of temperature and velocity, by assuming pressure fluctuations to be negligible. As in this case the cross-spectra of velocity and stagnation temperature have disappeared, the velocity spectrum can then be derived with a hypothesis of SRA for each frequency. The case of our experiments necessitates a more general hypothesis, since in the heated case fluctuations of total temperature cannot be neglected. However, it seems that the extra terms compensate one another, and it should be underlined that the same result still applies.

It is now examined if the various ranges of turbulence spectra have the same characteristics as in subsonic flows. In practice, scales derived from particular ranges (such as integral scales or production scales) will be considered with external and internal scalings and compared to the low speed results.

Firstly the inertial zone and the dissipative range are examined. An evaluation of the influence of density on Kolmogorov's scale is recalled (Debiève et al. [15]) to show the sensitivity of the high wave number limit to this parameter. In the logarithmic zone turbulence production $-\overline{u'v'}\frac{\partial U}{\partial y}$ balances the dissipation rate ε

$$-\overline{u'v'}\frac{\partial U}{\partial y} = \varepsilon,$$

and the shear stress is constant $\overline{\rho}\overline{u'v'} = \rho_w u_\tau^2$.

In supersonic layers, the law of the wall takes the form:

$$\frac{1}{u_\tau}\int_0^U \left(\frac{\overline{\rho}}{\rho_\mathrm{w}}\right)^{1/2} \mathrm{d}U = \frac{1}{\kappa}\ln y^+ + B.$$

The expression of dissipation becomes $\varepsilon = (\frac{\rho_\mathrm{w}}{\rho})^{3/2}\frac{u_\tau^4}{\nu_\mathrm{w}}\frac{1}{\kappa y^+}$.

Using a power law for the dynamic viscosity $\mu \propto T^m$, the Kolmogorov's scales of length and of velocity are derived:

$$\eta = \frac{\nu_\mathrm{w}}{u_\tau}\left(\kappa y^+\right)^{1/4}\left(\frac{\rho}{\rho_\mathrm{w}}\right)^{\frac{-6m-3}{8}},$$

$$\upsilon = u_\tau\left(\kappa y^+\right)^{-1/4}\left(\frac{\rho}{\rho_\mathrm{w}}\right)^{\frac{-2m-5}{8}}.$$

In the log zone, for the adiabatic case, the ratio ρ/ρ_w is generally less than 1.2. As a typical value of m is 0.75, the power $(-6m-3)/8$ is only 0.94, and the influence on η is about 20%. A first conclusion is that the limit imposed to the dissipative scales is practically inversely proportional to the density ratio, in the law of the wall, at the same ν_w/u_τ and y^+.

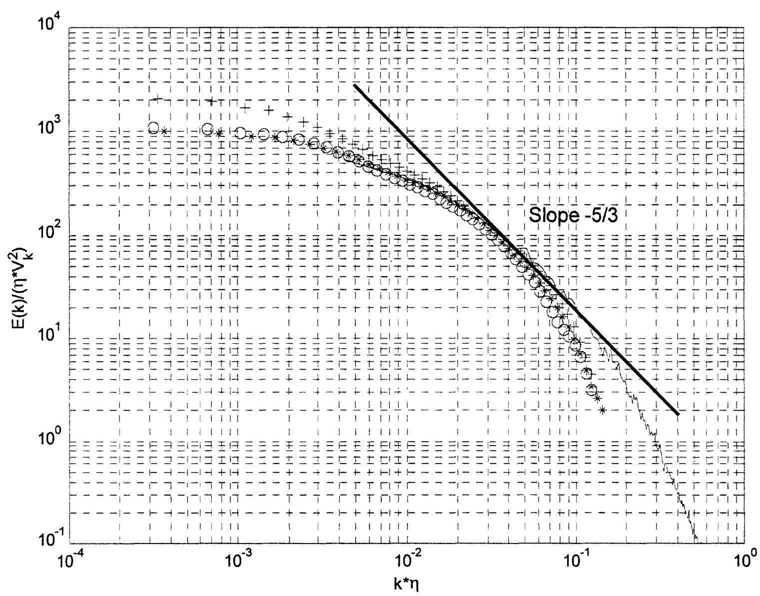

Figure 3. Power spectral densities in adiabatic boundary layers. ○ $y^+ = 36$; + $y^+ = 61$; $*$ $y^+ = 106$; ——— Antonia [17] ($M = 0$, $y^+ = 68$).

It is known that in subsonic flows matching requirements impose that a $-5/3$ law should exist on turbulence spectra in the log zone of a boundary layer (Perry, Henbest and Chong [16]). It was suggested in Smits and Dussauge [1], that for non-hypersonic boundary layers, Kolmogorov scaling can be applied to the high wave number range. Therefore, if the Kolmogorov scale is used to represent spectra, a similarity exists for high wave numbers, including the inertial and dissipative ranges. Such a result can be observed in *figure 3* where the spectra measured in a supersonic boundary layer on adiabatic plate are compared to a subsonic spectrum (Antonia [17]). These spectra were all measured in the log zone of the layers. Our measurements match the subsonic data mainly at the beginning of the inertial zone. As a matter of fact, only the start of the $-5/3$ law could be safely measured, because of the lack of resolution of the probe and of the apparatus. A rough estimation of the space integration of the wire was made from the work of Wyngaard [18]. η was estimated from the previous relations. This led to a correction on the spectrum of about 20% for $k\eta = 0.1$, and for $\eta/l \approx 0.4$. This correction is certainly not sufficient to restore the spectrum at the highest wave numbers, but it seems efficient to extend the similarity zone in the inertial range. As a first result, the Kolmogorov scaling and the inertial range seem to be very robust, and remain unaffected by these various conditions.

Secondly, an average scale of the turbulent perturbations, characterized by the integral scale is considered. The determination of the integral scale of the signal has been performed by the integration of the autocorrelation coefficient up to the first crossing to zero. Fluctuation diagrams on the scales of the hot wire signal have been made to determine the scales of velocity and temperature.

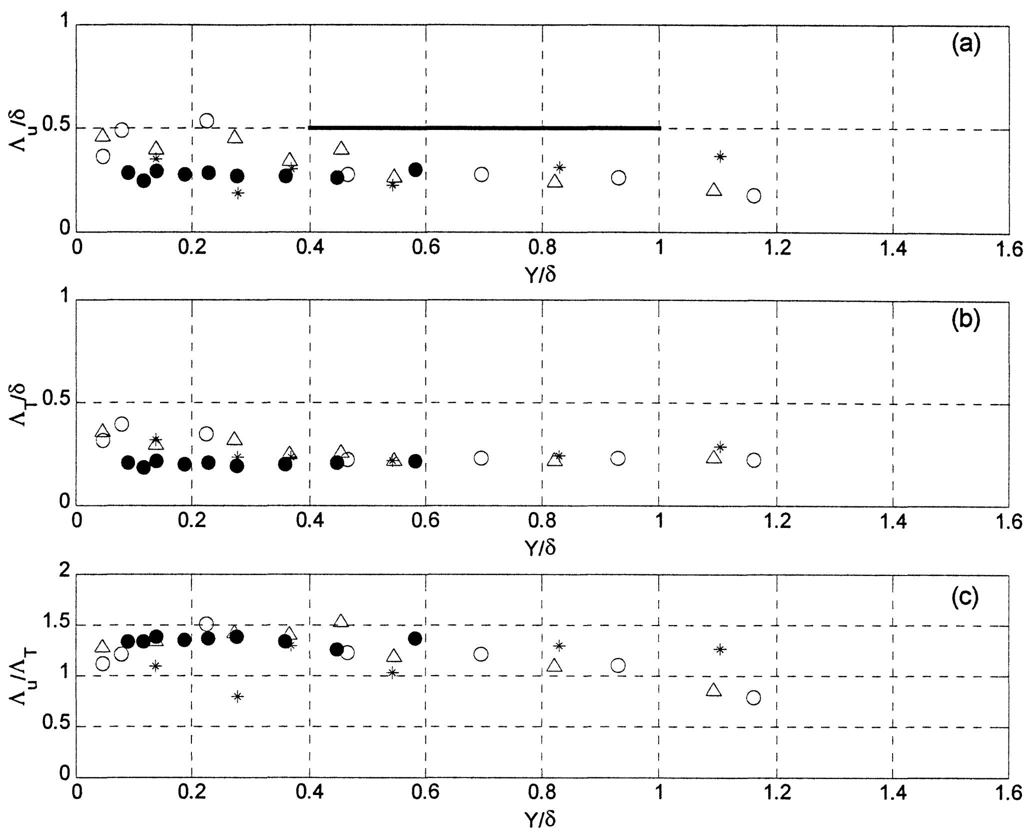

Figure 4. Integral scales in supersonic boundary layers: ● Audiffren [19]; ○ adiabatic, run 1; △ adiabatic, run 2; ∗ heated; ———— subsonic scale.

The profiles of the integral scales of velocity and temperature normalized by the boundary layer thickness are presented in *figure 4*. It can be remarked that these scales can be considered as constant in the external layer, $\Lambda_u \approx \delta/4$ for velocity, and about $\approx \delta/5$ for temperature. Their ratio (*figure 4(c)*), in agreement with the rest of the spectra, indicates that the time integral scale of temperature is smaller than the velocity scale. Closer to the wall, ($y/\delta \approx 0.2$ or 0.3) the integral scale is larger, and reaches values close to 0.4δ. The results of Audiffren [19] in the same boundary layer were obtained by determining the integral scale by measuring the low frequency part of the spectra, since $\Lambda_u = UE_u(0)/4$, $\Lambda_T = UE_T(0)/4$, where $E(f)$ is the spectrum normalized to unity. They agree well with the present determination in the external part of the layer. For $y/\delta \approx 0.2$, with some difference with the present results, Audiffren found that the scale keeps practically a constant value. The reason for this trend was that Audiffren determined the scale from the spectrum at zero frequency. In practice, she did not measure the very low frequency part of the spectrum, so that she had to extrapolate the spectrum to zero frequency to obtain the integral scale. Therefore, the contribution of a frequency band at very low frequency was neglected, which, near the wall, can be of higher level.

In the main part of the layer, the velocity scale is $\Lambda_u \approx \delta/4$. The heating of the supersonic layer, which increases the density ratio, does not change the value of the scale. Moreover it can be noted that the integral scale of velocity is significantly smaller than the typical subsonic value, $\Lambda_u \approx \delta/2$. This is in good agreement with a synthesis of results (Dussauge, Smits [8]) which shows the same trend: the integral scale decreases with increasing Mach number. This property is found to be independent of the heat flux.

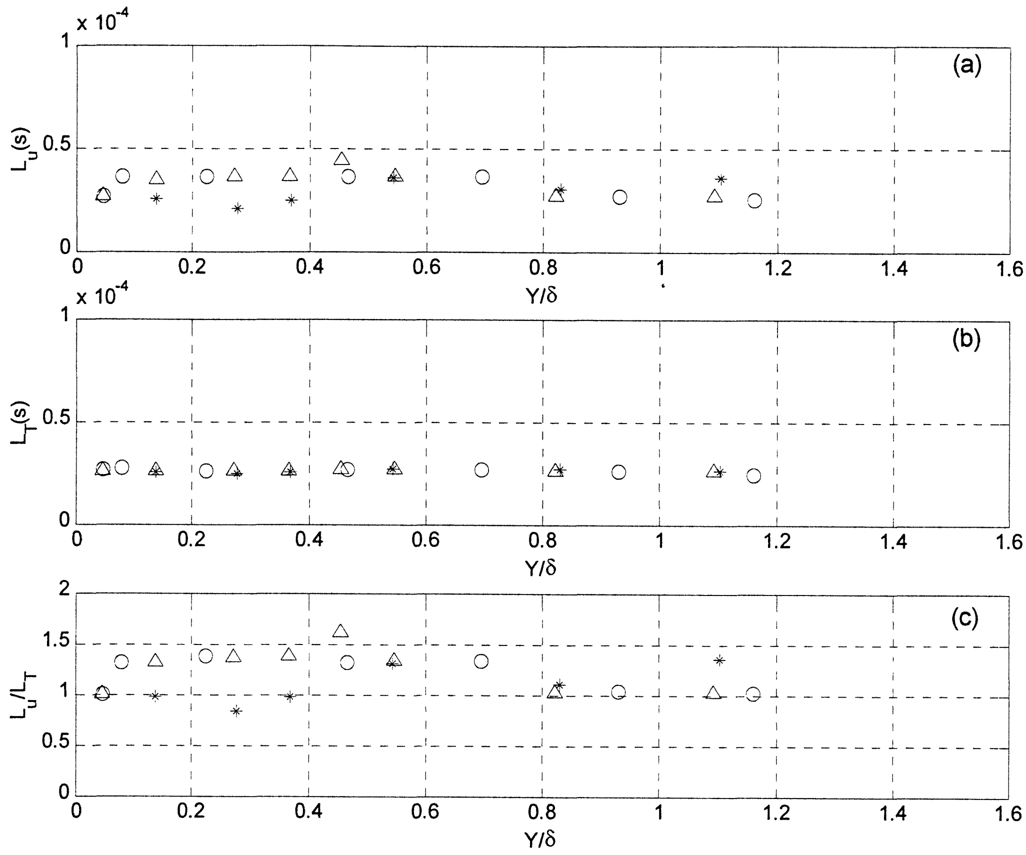

Figure 5. Production scale in supersonic boundary layers: ○ adiabatic, run 1; △ adiabatic, run 2; $*$ heated; ———— subsonic scale.

Thirdly, energetic scales, characterized by the production range are now examined. In this range, the frequency spectrum varies like f^{-1} (see for example Hinze [20]) and the product $fE(f)$ is constant. In practice, it is difficult to find a range of large extent. Generally, the product $fE(f)$ shows a maximum for a given frequency. This particular frequency will be defined as the production scale. In our case, this scale splits the spectrum into two sub-domains of roughly comparable energies.

The production scales of time for velocity and temperature L_{u} and L_{T} are presented in *figure 5*. They are almost constant in the whole layer, even near the wall for distances $y/\delta = 0.05$. The ratio $L_{\mathrm{u}}/L_{\mathrm{T}}$ is larger than 1 (*figure 5(c)*) for adiabatic and heated cases. Using Taylor's hypothesis, it is found that, for the adiabatic and heated plates, $UL_{\mathrm{u}}/\delta \approx 1.4$ in the external part of the flow. This corresponds to scales smaller than in subsonic

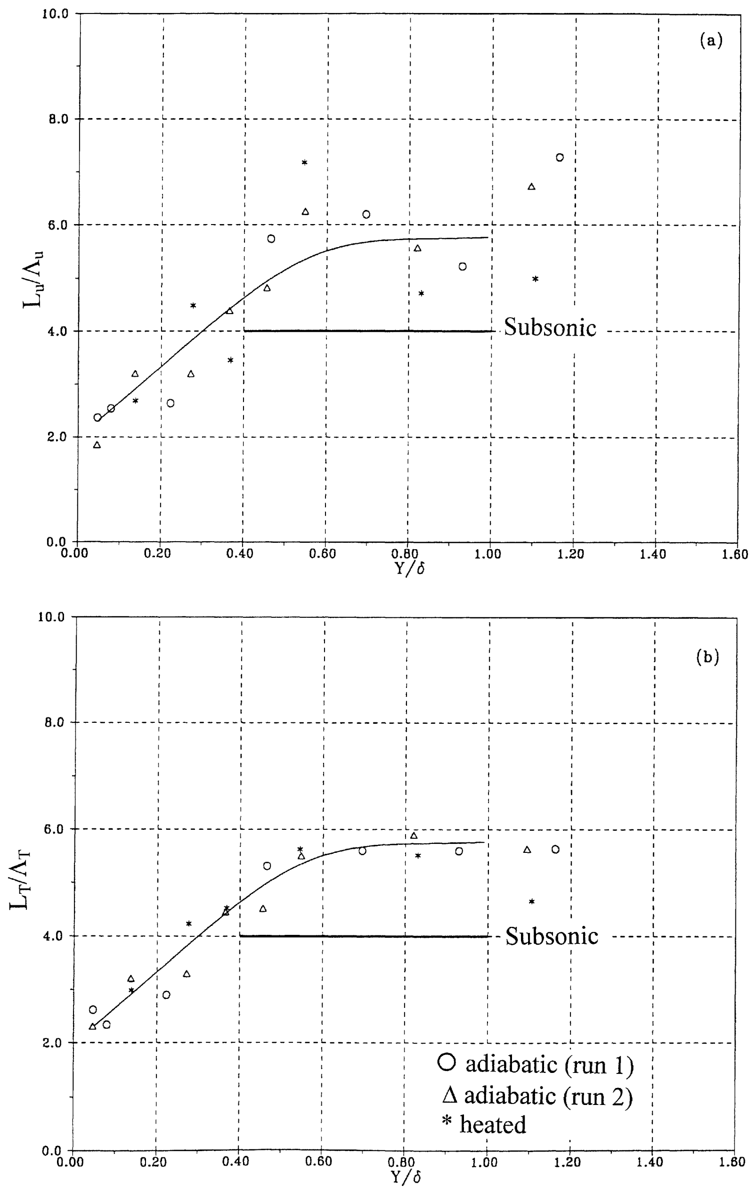

Figure 6. Ratio of integral and production scales for velocity and temperature in boundary layers.

flow, where $UL_u/\delta \approx 2$. Again, this is in agreement with the observations of Dussauge and Smits [8], who found $0.8 < UL_u/\delta < 1.4$ in a large range of Reynolds and Mach numbers, in adiabatic conditions.

The shape of the spectrum in the energetic range can be characterized by the ratio L/Λ (*figure 6*) This ratio is not constant in a section: this shows a modification of the shape of the spectra through the layer. This behavior is about the same for temperature and velocity. On the other hand, the comparison between subsonic and supersonic flows for the external layer puts in evidence an important effect of Mach number which has no equivalent when changing the density ratio by wall heating.

To sum up these results derived from the analysis of spectra for velocity, mass flux, static and total temperature, it is verified that in spite of some shifts, the frequency ranges for these quantities are comparable for adiabatic or heated wall conditions. As far as comparisons with subsonic flows are concerned, the supersonic regime involves smaller energetic scales. Changing the density ratio by heating the wall brings no evidence of the alteration of the large scales. An illustration of the modification of the spectrum shape inferred from the present results is sketched in *figure 7*, which gives a simplified comparison of typical subsonic and supersonic situations. For high wave numbers, the limit given by Kolmogorov's scale depends on density ratio, and would correspond to a decrease of η in adiabatic and heated flows. At low frequencies, the level of the spectrum is divided by a factor about 2. The point of slope –1 is shifted to frequencies twice as large as their subsonic counterpart, when external scaling is used. The part in $-5/3$ power law has to match this energetic range (shifted to higher frequencies) with the dissipative range, which, in adiabatic and heated layers, involves scales smaller than in subsonic flow.

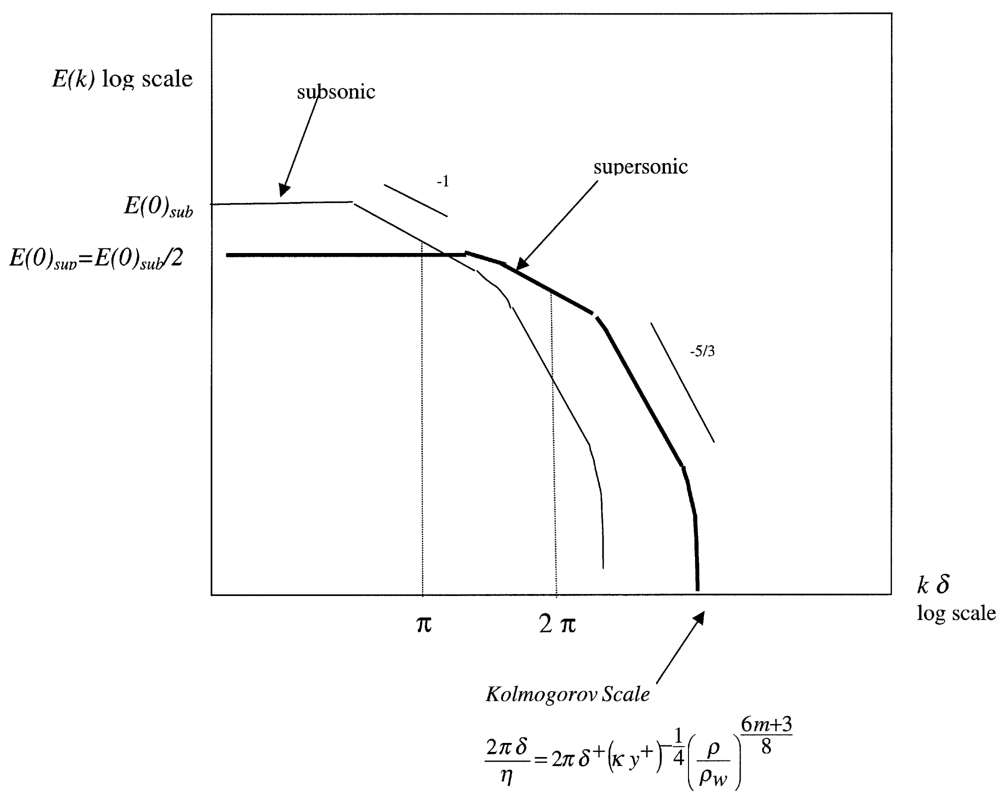

Figure 7. Sketch of spectra in subsonic and supersonic boundary layers.

3. Analogies and acoustic radiation in a supersonic mixing layer

We can now examine the case of a mixing layer flow at a convective Mach number $M_c = (a_2 U_1 + a_1 U_2)/(a_1 + a_2) = 0.61$ (indices 1 and 2 refer respectively to the external flows of high and low velocity, see the end of section 3), in which the effect of compressibility becomes important, since a reduction of 25% of the spreading rate is observed. Moreover, for this value of M_c, it has been found that the large eddies become three-dimensional (Clemens and Mungal [21]). Such a flow was installed in the supersonic wind tunnel of IRPHE. The configuration considered here is the same as in Barre, Quine, Dussauge [22], with some modifications. Firstly, arrangements were made in the diffuser to extend the measurements up to a distance of 300 mm from the trailing edge of the splitter plate (instead of 200 mm in Barre et al. [22]), i.e. about 10 times the layer thickness further downstream. Secondly, the probe holder system was replaced by a pure $X-Y$ system. This was done by placing the stepping motors and the railing system in a pressurized box fixed on the ceiling of the test section. A groove was machined in the upper wall of the test section to place the mast, which holds the probes. The pressurized box was set at a pressure close to the pressure in the test-section, and two rubber joints were mounted along the edges of the groove, to minimize possible recirculating flows around the probe holder.

Mixing layers are known to be very sensitive to boundary conditions, including the downstream conditions. A consequence of a modification in the design of the downstream part of the test section, and of the probe holder system is that the static pressure distribution is not exactly the same as in Barre et al. [22]. Therefore, a new exploration of the mean field was performed, with essentially the same conventional methods as in the previous reference (measurements of static and pitot pressure and of stagnation temperature by thermocouple), but in greater details, and by including new sections downstream. Technical details of the arrangements and rough data profiles can be found in Menaa [23]. In the following results, the origin of the frame of reference is taken at the trailing edge of the separating plate. The longitudinal coordinate is x, the vertical one is y; y is taken positive towards the supersonic external flow. Although the incoming conditions are the same as in Quine [24], small differences have appeared. A rather uniform distribution of pressure was obtained for a mass flux in the subsonic flow producing a nominal Mach number M_2 of 0.35. The nominal Mach number on the supersonic side is $M_1 = 1.85$. The resulting nominal convective Mach number is 0.61 instead of 0.62 in Barre et al. [22].

A collapse of the velocity profiles showing similarity is observed for $x > 140$ mm. The spatial growth of the layer is shown in *figure 8*. Two definitions of the thickness are used: the definition of the thickness δ_s given at the Stanford Conference (the thickness is defined between the points where $U = U_2 + 0.95(U_1 - U_2)$ and $U = U_2 + 0.316(U_1 - U_2)$, and the vorticity thickness $\delta_\omega = \Delta U/(\partial U/\partial y)_{\max}$. The solid line is obtained by assuming that for a subsonic layer with zero velocity ratio and constant density, $(\mathrm{d}\delta_s/\mathrm{d}x)_0 = 0.115$, and $(\mathrm{d}\delta_\omega/\mathrm{d}x)_0 = 0.15$. The formula proposed by Papamoschou and Roshko [5] to derive the spreading rate of a layer with velocity ratio $r = U_2/U_1$, density ratio $s = \rho_2/\rho_1$ and convective Mach number M_c was employed:

$$\frac{\mathrm{d}\delta}{\mathrm{d}x} = \frac{1}{2}\left(\frac{\mathrm{d}\delta}{\mathrm{d}x}\right)_0 \frac{(1+r)(1+s^{1/2})}{1+rs^{1/2}}\Phi(M_c).$$

The definition of $\Phi(M_c)$ is the same as in Papamoschou and Roshko [5]. The values of $\Phi(M_c)$ used in the present work are the values reassessed in Smits and Dussauge [1]. The agreement with the measurements is very good, and a linear growth is observed in the sections in which the mean velocity profiles are self-similar.

Mean flow measurements were detailed enough to determine turbulent friction and heat flux. Turbulent friction was determined from the integration of the momentum equation. In order to limit the scatter in the

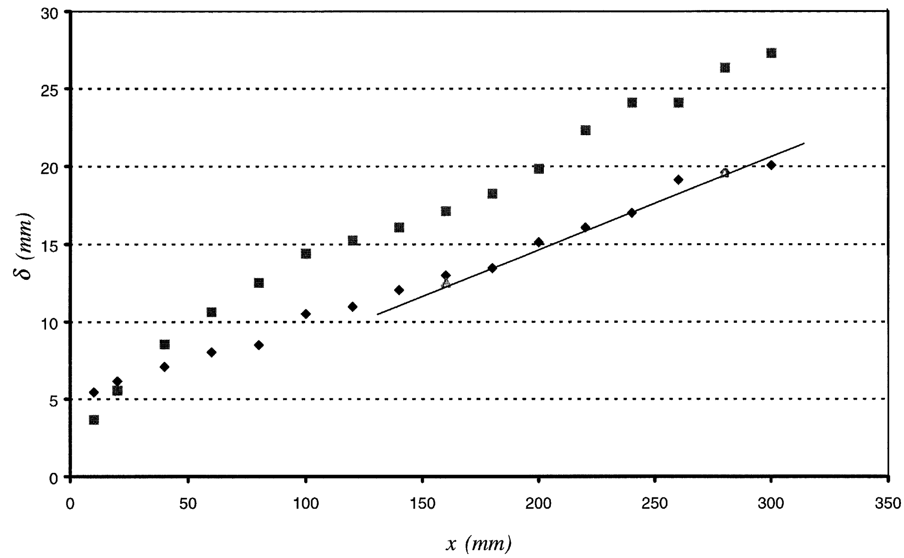

Figure 8. Mixing layer thickness: −Δ− Langley; ♦ Stanford definition; ■ vorticity thickness.

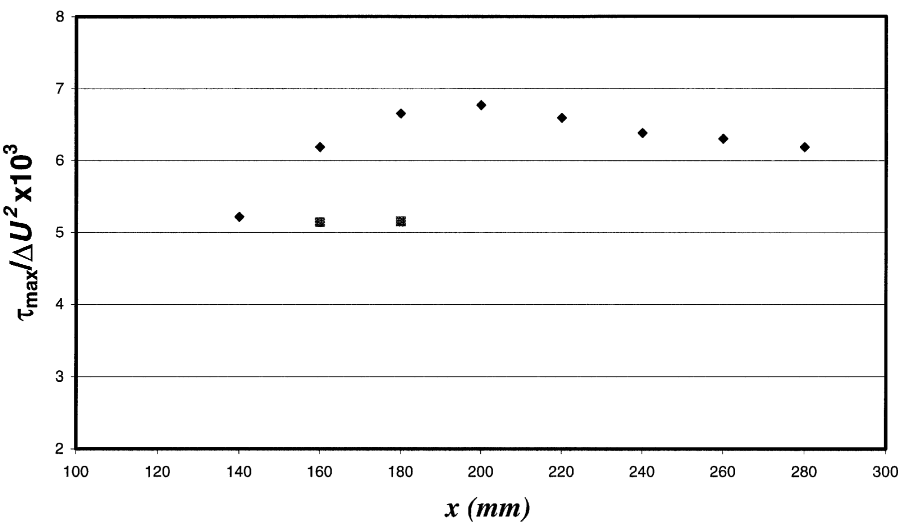

Figure 9. Longitudinal evolution of maximum friction (Mixing layer, $M_c = 0.61$): ♦ present data; ■ Quine [24].

determination from the experimental data, the following formulation was retained:

$$\tau = \left(1 - U^+\right)\frac{\mathrm{d}}{\mathrm{d}x}\int_{\delta_2}^{y} \overline{\rho}U(U - U_2)\,\mathrm{d}y + U^+\frac{\mathrm{d}}{\mathrm{d}x}\int_{\delta_1}^{y} \overline{\rho}U(U - U_1)\,\mathrm{d}y,$$

where $U^+ = (U - U_2)/(U_1 - U_2)$; δ_1 and δ_2 are respectively the edges of the mixing layer on the high velocity side and on the low velocity side and U is the Favre average of the velocity. It was found that integrating the profiles with respect to y before differentiating generates less scatter than the direct method, in which the order of the integration and derivation is inverted. Q, the flux of total temperature was deduced in the same way from the integration of mean total enthalpy equation:

$$Q = -\overline{\rho v' h_t'} = (1 + H^+) \frac{\mathrm{d}}{\mathrm{d}x} \int_{\delta_2}^{y} \overline{\rho} U (h_t - h_{t2})\, \mathrm{d}y + H^+ \frac{\mathrm{d}}{\mathrm{d}x} \int_{\delta_1}^{y} \overline{\rho} U (h_t - h_{t1})\, \mathrm{d}y,$$

where v' is the vertical velocity fluctuation, h_t is the Favre average of the total enthalpy, h_t' the fluctuation and $H^+ = (h_t - h_{t2})/(h_{t1} - h_{t2})$.

The heat flux was derived from the total enthalpy flux from the relation:

$$\overline{\rho v' T'} = (U\tau - Q)/C_p.$$

The evolution of the maximum of turbulent friction is given in *figure 9*, and the profiles in *figure 10*. The same quantities for the heat flux are shown in *figures 11* and *12*. In *figure 9*, the value of the maximum friction obtained by Quine in the previous configuration of the flow is indicated. The difference with the present determination is of 20%. It is felt that this discrepancy reflects in some way the differences between the flows, but remains also of the order of the accuracy in the determination of friction from mean field measurements. In the present determination, the maximum value is almost constant for $x > 200$ mm, and the resulting value is in good agreement with the compilation of friction given in Barre et al. [22] and in Smits and Dussauge [1]. In our knowledge, no other determination of the heat flux is available in the literature. It is not clear if the heat flux has reached an asymptotic value. However, in the developed part of the flow, the variations are not large from a section to another one, and are only of 20%.

The turbulent Prandtl number $Pr_\mathrm{t} = (-\overline{u'v'}\partial T/\partial y)/(-\overline{T'v'}\partial U/\partial y)$ and the ratio between fluxes have been determined. If Pr_t is close to 1, it is expected that the relation derived from the Strong Reynolds Analogy (SRA) holds:

$$\frac{\overline{v'T'}/T}{(\gamma - 1)M^2 \overline{u'v'}/U} = \frac{\overline{v'T'}}{C_\mathrm{p} \overline{u'v'} \overline{U}} \approx 1.$$

The results are given in *figures 13* and *14*. The turbulent Prandtl number takes values between 0.7 and 0.8, which are expected in subsonic free shear flows. In the last sections, a value close to 0.8 is found in the middle of the layer. The SRA relationship is rather well verified. This is more surprising: Smits and Dussauge [1] have shown that in shear layers, where the Pr_t can deviate significantly from 1, for example for $Pr_\mathrm{t} = 0.6$, the SRA relation is not well verified. Indeed, in the present results, the SRA holds in the last section where $0.7 < Pr_\mathrm{t} < 0.8$.

A first conclusion can be drawn. At $M_\mathrm{c} = 0.61$, although compressibility acts significantly on the spreading rate, and although the shape of the eddies is altered, the turbulent Prandtl number is unchanged and the Reynolds analogy between heat- and momentum transfers appears to be as valid as in supersonic boundary layers where the effects of compressibility are weak.

Another key point for compressible turbulent flows is the importance of the energy loss by acoustic radiation. Barre et al. [22] have measured the intensity of the pressure radiated by their mixing layer into the supersonic external flow; they have also measured the speed of the sound sources. However, they have not evaluated the energy radiated by acoustic waves, although their data can provide an assessment of the importance of such phenomena, as J. Laufer [25] did for turbulent boundary layers. It was checked in Muscat [10] that the pressure fluctuations in the outer flow in the new configuration are in good agreement with the measurements of Barre et al. [22]. It is deduced that these results can be used for evaluating the acoustic losses in the new mixing layer at a nominal Mach number of 0.61.

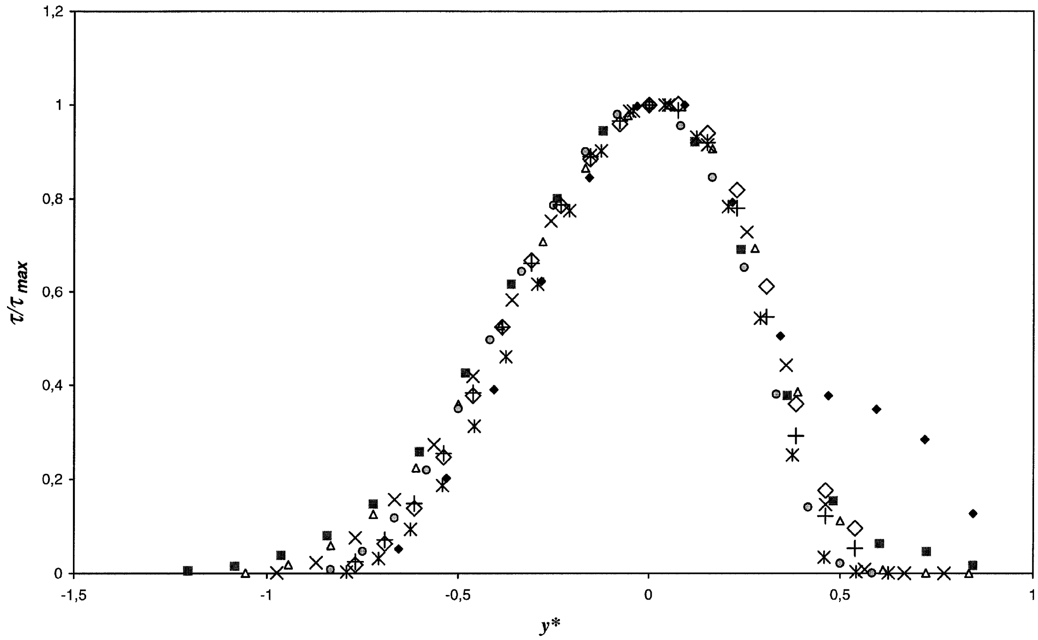

Figure 10. Dimensionless turbulent friction profiles (Mixing layer, $M_C = 0.61$): ♦ $x = 140$ mm; ■ $x = 160$ mm; △ $x = 180$ mm; × $x = 200$ mm; ∗ $x = 220$ mm; ○ $x = 240$ mm; + $x = 260$ mm; • $x = 280$ mm.

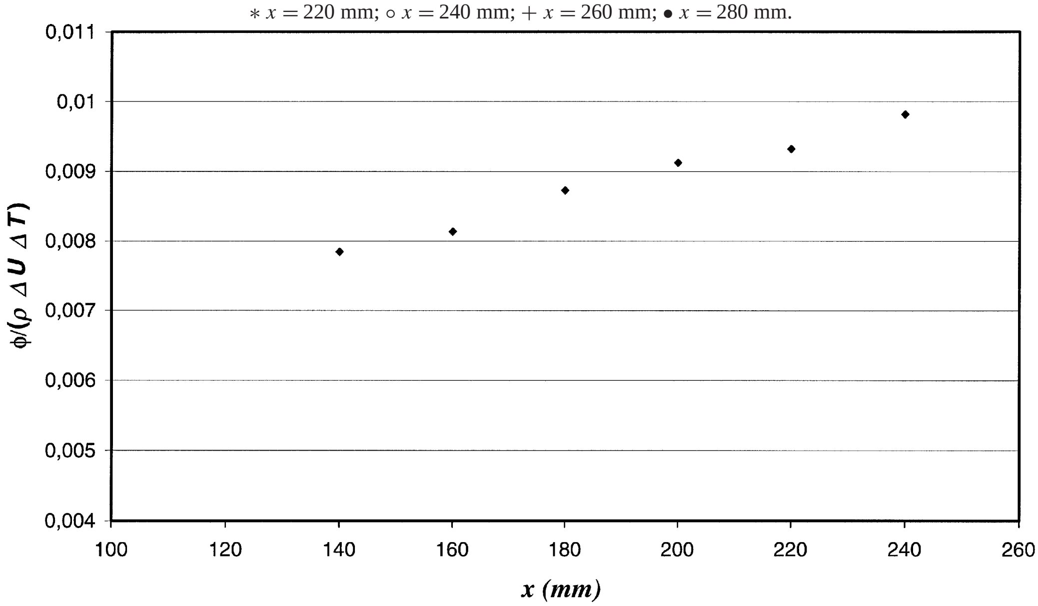

Figure 11. Longitudinal evolution of maximum heat flux (Mixing layer, $M_C = 0.61$).

The measurements have shown that the velocity difference $U_1 - U_s$ between the supersonic stream and the noise sources is supersonic. Hence, it can be assumed the pressure waves are predominantly Mach waves, and the characteristics of the energy flux can be derived from the usual relations for isentropic fluctuations. The energy flux per unit surface is usually called the acoustic intensity. This vector quantity is normal to the wave

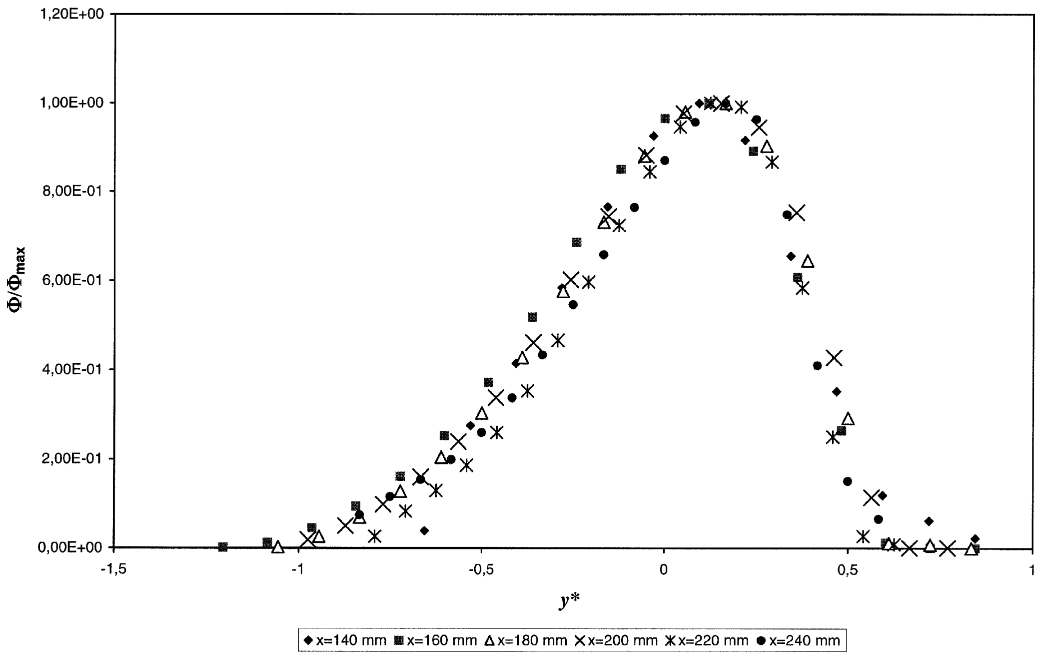

Figure 12. Dimensionless turbulent heat flux profiles (Mixing layer, $M_c = 0.61$): ♦ $x = 140$ mm; ■ $x = 160$ mm; Δ $x = 180$ mm; $\times$ $x = 200$ mm; $*$ $x = 220$ mm; • $x = 240$ mm.

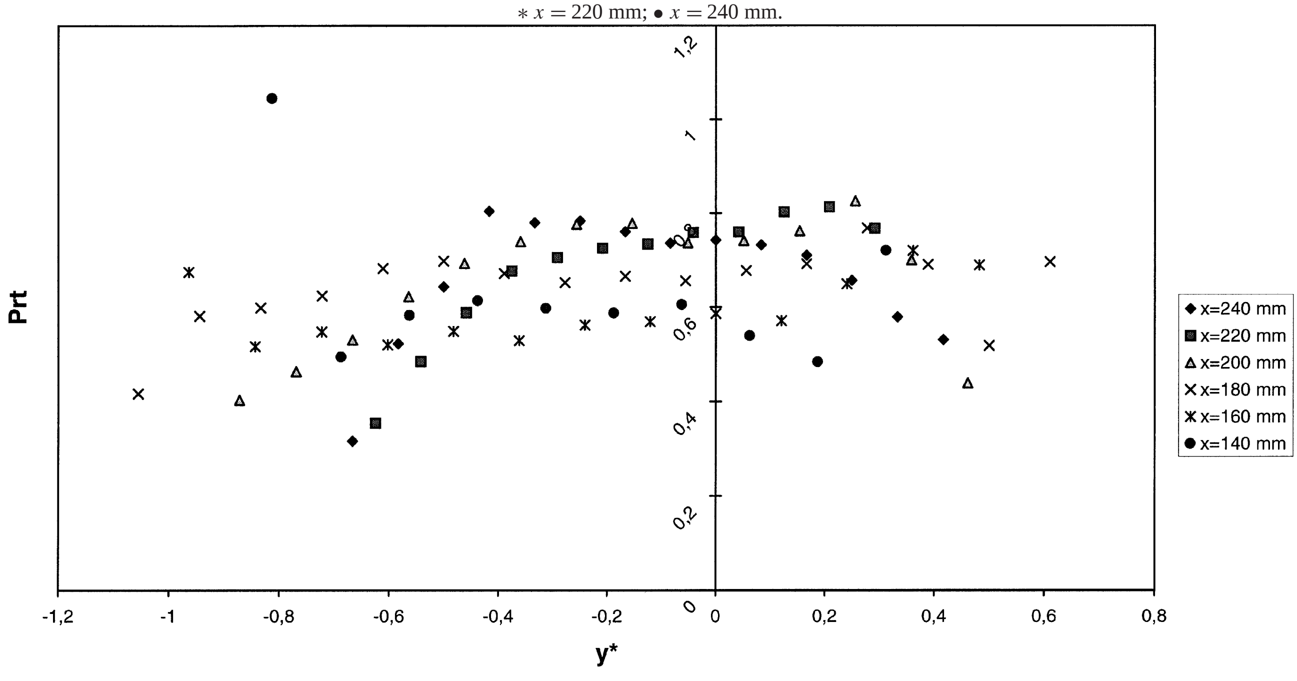

Figure 13. Turbulent Prandtl number (Mixing layer, $M_c = 0.61$).

front, and its modulus is:

$$I = \frac{\overline{p'^2}}{\rho a}.$$

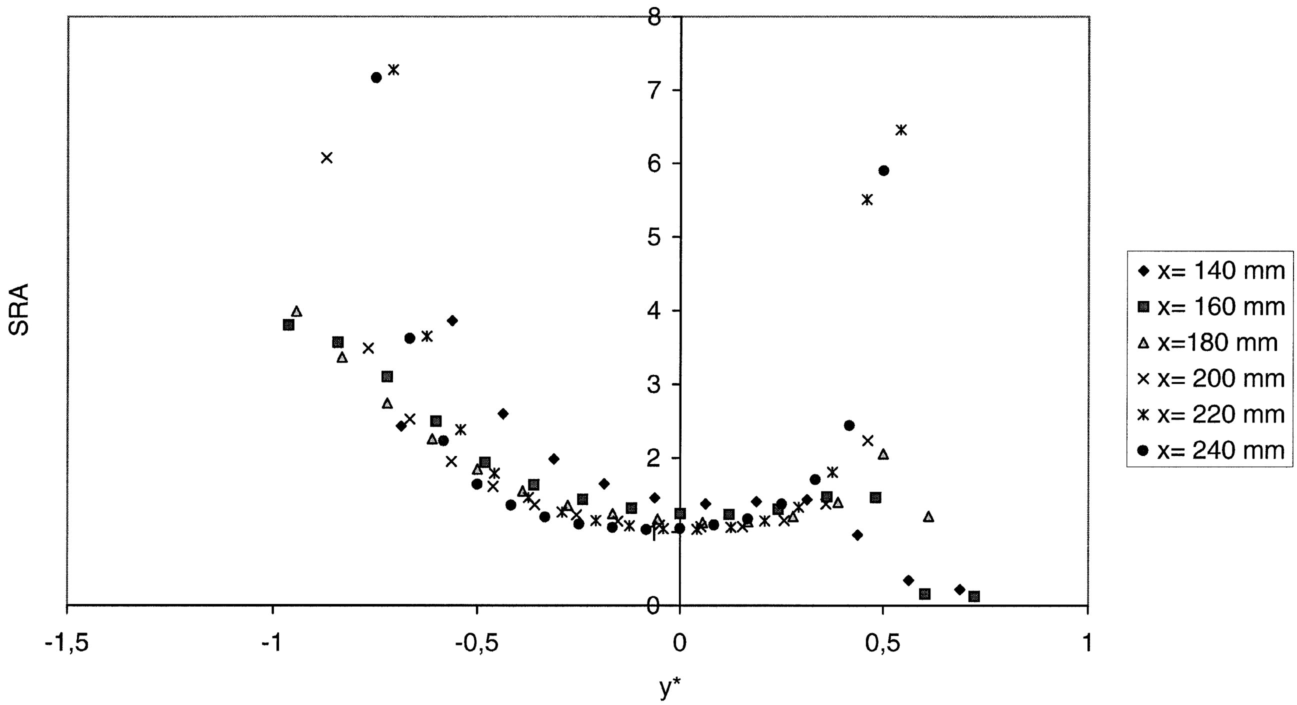

Figure 14. Strong Reynolds analogy in the mixing layer.

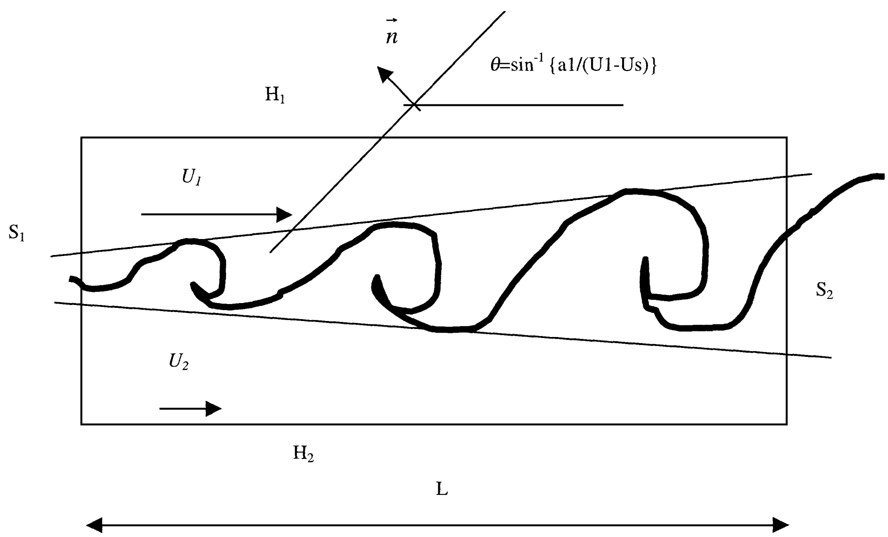

Figure 15. Volume of control for the budget of kinetic energy.

A question is to determine to which turbulent quantity the acoustic intensity should be compared. The simplest is probably to consider the equation for turbulent kinetic energy, and to integrate it over a control volume, (*figure 15*) defined by two sections $S1$ and $S2$, two horizontal planes $H1$ and $H2$; its dimension is unity in the spanwise direction.

The equation of turbulent kinetic energy reads, for Favre averaged variables:

$$\frac{\partial}{\partial x_j}\overline{\rho} k \tilde{U}_j = P + D - \overline{\rho}\varepsilon,$$

where $k = \frac{1}{2}\overline{\rho u'_j u'_j}/\overline{\rho}$, $\tilde{U}_j$ are the components of Favre averaged velocity, ρ is the mean density, the overbar denotes an average, P is the turbulence production, $D = \frac{\partial}{\partial x_j}(\overline{\rho u'_i u'_i u'_j} + \overline{p' u'_j})$ is the term of turbulent diffusion, ε the rate of dissipation per unit mass, p' is the pressure fluctuation, and u'_j the components of velocity fluctuation. Integrating over the volume V bounded by the surface S yields:

$$\int_V \left(\frac{\partial}{\partial x_j}(\overline{\rho} k \tilde{U}_j)\right) \mathrm{d}\, vol = \int_V P \,\mathrm{d}\, vol - \int_S (\overline{\rho u'_i u'_i u'_j} + \overline{p' u'_j}) n_j \,\mathrm{d}S - \int_V \varepsilon \,\mathrm{d}\, vol.$$

As we are interested by the energy flux radiated into the outer flows, we will consider only surface H_1 and H_2. We calculate now the flux through H_1, whose normal vector reduces to n_2. As H_1 is chosen far from the edges of the mixing layer, the fluctuations are only of acoustic nature. A consequence is that $\overline{p' u'_2}$ is large compared to $\overline{\rho u'_i u'_i u'_j}$. Now the component u'_2 is related to p' through the classical isentropic relation:

$$p' = \frac{\rho a u'_2}{\sin\theta} \quad \text{and} \quad \overline{p' u'_2} = \frac{\overline{p'^2}}{\rho a}\sin\theta,$$

where θ is the angle between the Mach wave and the plane H_1 (see *figure 15*). Therefore, the surface integral can be expressed as $\int_{H_1} \frac{\overline{p'^2}}{\rho a}\sin\theta \,\mathrm{d}S$.

This shows that the flux of acoustic energy through H_1 has to be compared to the volume integral of the source terms of the equation for k, for example to the volume integral of the production term. In an alternate and simpler way, the apparent acoustic intensity $\overline{p' u'_2} = \frac{\overline{p'^2}}{\rho a}\sin\theta$ is to be compared to the integral of turbulence production in a section.

This method was applied to the measurements of Barre et al. [22]. The measurements were performed only in the supersonic external flow. It was assumed that the energy loss is symmetric, i.e. that the amounts of energy lost by acoustic radiation are the same into either of the external flows. The angle θ has to be determined. The velocity U_s of the acoustic sources radiating into the supersonic external flow was measured by Barre et al. [22]. The angle θ is, by definition, such as $\cos\theta = -\frac{a_1}{(U_1 - U_\mathrm{s})} = -\frac{1}{M_\mathrm{s}}$; finally the power involved by acoustic emission is estimated as:

$$P_\mathrm{a} = 2\frac{\overline{p'^2}}{\rho a}\frac{\sqrt{M_\mathrm{s}^2 - 1}}{M_\mathrm{s}}.$$

Using the measurements of Barre et al. in which $M_\mathrm{s} \approx 1.4$, and $\overline{p'^2}/p^2 \approx 4 \times 10^{-4}$, it is found that $P_\mathrm{a} \approx 1100\ \mathrm{W/m^2}$, while the integral of production at section $x = 200$ mm is $3.2 \times 10^4\ \mathrm{W/m^2}$. Therefore, the acoustic loss of energy is only 3% of the production terms. This level is of course weak. It must be underlined that the value measured by Barre et al. for the acoustic intensity, is very conservative. As their measurements were performed in the test section of a nozzle, they measured the noise radiated by the mixing layer plus the noise emitted by the wall boundary layers. From their data, the pressure fluctuations produced by the boundary layers have a rms value $p'_\mathrm{rms}/p \approx 0.8 \times 10^{-2}$. If the pressure fluctuations resulting from the different sources are independent, the variances can be subtracted, and this would give an estimate of the acoustic loss 15% lower, of about 2.5% of the production term. The spreading rate, as shown in Vreman et al. [6], can be related to the integral of turbulence production in a section. It is therefore very likely that for at a convective Mach number of 0.61, the acoustic flux is too weak to explain the reduction of 30% observed on the spreading rate.

4. Discussion and concluding remarks

The objective of this paper was to give an assessment of the influence of density variations and of acoustic phenomena, in some typical shear flows, at supersonic speeds. The first point on boundary layers described as flows with variable properties (see for example Spina et al. [2]) is very correct for the links between velocity and shear stress. This does not hold, however, for turbulence scales: the changes in the energetic part of the spectrum are not caused by density variations. Therefore, this is a consequence of the high speeds, and an effect of some Mach number. In these boundary layers, the shape of the turbulence signal (and then probably the repartition of velocity fluctuations in space) is modified, but transport processes can be represented as in low speed flows. This modification of the shape of the signal may be attributed either to a change in the shape of the large eddies, or to a modification of the flow field, which they produce. It was guessed in Smits and Dussauge [1] and in Debiève et al. [15], that the 'inactive motions' which have some energy, but produce no significant shear stress, and are, in some way, related to the field induced by the eddies, could be enhanced in the supersonic regime. This is consistent with the present results, for which the heating is of little importance for the global dynamics of the layer (Debiève et al. [9]), and therefore would produce little modification to the inactive motions. This supports the idea that modifications of the scales relative to u' are the first signs of the influence of compressibility, which needs higher Mach numbers to become fully evident.

On the other hand, when compressibility effects are present, a likely expectation is to find a larger influence of acoustics. The present results show very clearly that for $M_c = 0.61$, the acoustic contribution is not strong enough to alter traditional schemes of turbulent transport. This suggests that, in spite of a reduction of mass entrainment and of a modification of the characteristic scales of space, new phenomena, if any, are not sufficiently developed to modify phenomenological representations of the turbulent fluxes. Similar experiments at higher Mach numbers are necessary and would be very informative for this point. The evaluation of the energy loss by acoustic radiation has been given. The result of this experimental verification is in agreement with the common idea that acoustics contains little energy, even if the level of noise is high. Again, experiments at higher Mach numbers would be necessary to determine critical values of M_c for this effect. Finally, the action of compressibility appears to be clearly distinct from the influence of density gradients, but it seems to be very intricate. If a modification of the structure of pressure is the cause of the trends observed in experiments, the first consequence is not in the enhancement of the far field, but in an alteration of pressure in the flow itself.

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